SULIT*

950/1, 954/1

STPM

MATHEMATICS T/S

PAPER 1

Three Hours (Tiga

JABATAN PELAJARAN NEGERI SABAH

EXCEL II TINGKATAN 6 ATAS
AUGUST 2010

SIJIL TINGGI PERSEKOLAHAN MALAYSIA

This question paper consists of 5 printed pages.
(Kertas soalan ini terdiri daripada 5 halaman bercetak.)

1 Given that $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & \frac{1}{2} & -3 \\ -4 & 0 & k^2 \end{pmatrix}$. Show that A is a non-singular matrix for all real values of $\boldsymbol{k}$. [4 *marks*]

2 Use the trapezium rule with integrals of width 0.5 to find an approximation for

$$\int_1^{2.5} \frac{1}{1+\ln x}\,dx$$

giving your answer correct to 2 decimal places. [4 *marks*]

3 Given α and β are the roots of the equation $x^2 - 28x + 16 = 0$. Obtain a quadratic equation whose roots are $\sqrt{\alpha}$ and $\sqrt{\beta}$. [5 *marks*]

4 Given that $Re(w)=1$ and $Re\left(\frac{1}{w}\right)=\frac{1}{4}$. Find all the possible complex numbers of w.

[6 *marks*]

5 The equation of a curve is $x^3 + xy + 2y^3 = p$, where p is a constant.

Find $\frac{dy}{dx}$ in terms of x and y. [3 *marks*]

It is given that the curve has a tangent which is parallel to the y-axis. Show that the y coordinate of the point of contact of the tangent with the curve must satisfy

$$216y^6 + 4y^3 + p = 0$$ [3 *marks*]

Hence, show that $p \le \frac{1}{54}$. [2 *marks*]

6 The function f is defined by

$$f(x) = x + \frac{1}{x}, \quad x \ge 1$$

(*a*) Show that $f(x)$ increases as x increases [3 *marks*]

(*b*) State the range of f [1 *marks*]

(*c*) Find an expression for $f^{-1}(x)$ [4 *marks*]

7 Given a polynomial function $P(x) = x^3 + 2x^2 - 3x - 6$,

(a) show that $x + 2$ is a factor of $x^3 + 2x^2 - 3x - 6$, [2 *marks*]

(b) find the other two linear factors of this polynomial. [3 *marks*]

(c) hence, solve the inequality $\dfrac{x^3 + 2x^2 - 3x - 6}{x - 1} \leq 0$. [3 *marks*]

8 Matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are given as $\boldsymbol{A} = \begin{pmatrix} 2 & 1 & -2 \\ -1 & 1 & -3 \\ -3 & -2 & 2 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 4 & -2 & 1 \\ -11 & 2 & -8 \\ -5 & -1 & -3 \end{pmatrix}$.

Find $\boldsymbol{AB}$ and deduce $\boldsymbol{A}^{-1}$. [4 *marks*]

Hence, express the following simultaneous equations as a matrix equation and solve the system of linear equations

$$2\boldsymbol{x} + \boldsymbol{y} - 2\boldsymbol{z} = 3$$

$$-2\boldsymbol{x} + 2\boldsymbol{y} - 6\boldsymbol{z} = -14$$

$$-3\boldsymbol{x} - 2\boldsymbol{y} + 2\boldsymbol{z} = -5.$$

[5 *marks*]

9 Prove that $9x^2 + 4y^2 - 18x + 16y - 119 = 0$ is an ellipse [5 *marks*]

Hence, sketch the graph of $9x^2 + 4y^2 - 18x + 16y - 119 = 0$ [4 *marks*]

10 A curve is given parameterically by the equations

$$x = 2 + t; \quad y = 1 - t^2$$

Show that the normal at the point with parameter t has equations

$$x - 2ty = 2t^3 - t + 2$$

[4 *marks*]

The normal at the point T, where $t = 2$ cuts the curve again at the point P, where $t = p$. Show that $4p^2 + p - 18 = 0$ and hence deduce the coordinates of P. [5 *marks*]

Find the cartesian equation of the curve and hence sketch the curve. [3 *marks*]

11 (a) Expand $(1+x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3.

Using $x=\frac{1}{8}$, find the approximate value of $\sqrt{2}$ correct to 5 decimal places. [5 *marks*]

(b) Express $\dfrac{1}{4r^2-1}$ in partial fractions. [4 *marks*]

Hence, find (i) $\displaystyle\sum_{r=1}^{n}\frac{1}{4r^2-1}$ [3 *marks*]

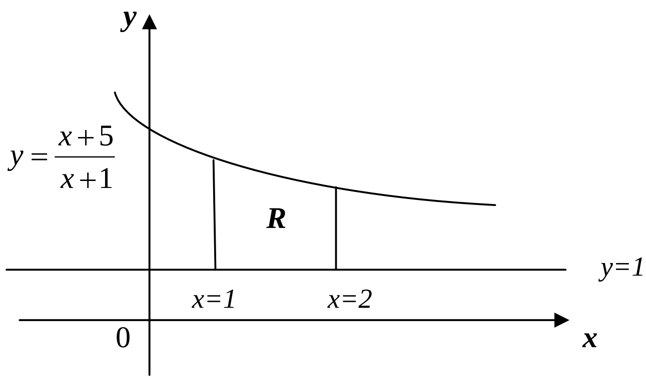
(ii) $\displaystyle\sum_{r=1}^{\infty}\frac{1}{4r^2-1}$ [1 *marks*]

12 (a) Given a curve $y=2+x-x^2$ and a straight line $y=-x-1$,

(i) sketch on the same coordinates axes, the curve and the straight lines, [2 *marks*]

(ii) determine the coordinates of their points of intersection, [2 *marks*]

(iii) calculate the area of the region bounded by the curve and the straight line. [4 *marks*]



(b) The region ***R*** shown in the diagram above is bounded by the curve $y=\dfrac{x+5}{x+1}$, the straight lines x = 1 and x = 2. Calculate the volume of the solid formed when the area is rotated through 2π radian about the straight line y = 1. [6 *marks*]

STPM 950/1, 954/1 (EXCEL 2010 JPNSabah)
Marking Scheme MATHEMATICS T & S (Paper1)

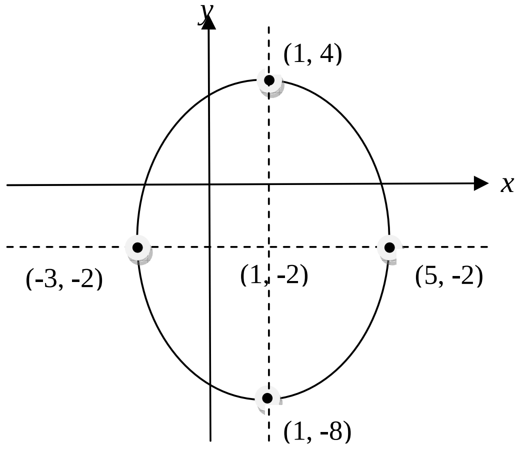
<table>
<tr><th>No.</th><th>Answer Scheme</th><th>Marks</th></tr>
<tr><td>1</td><td>
$$A=\begin{pmatrix}1&0&1\\0&\frac{1}{2}&-3\\-4&0&k^2\end{pmatrix}.$$
$$|A|=\begin{vmatrix}\frac{1}{2}&-3\\0&k^2\end{vmatrix}-0+\begin{vmatrix}0&\frac{1}{2}\\-4&0\end{vmatrix}$$
$$=\frac{1}{2}k^2+2$$
since $k^2\geq 0$, $\forall k\in\Re$
hence $\frac{1}{2}k^2+2\neq 0$ } either one statement to get M1
A^{-1} exists,
∴ A is a non-singular matrix } Both statements
</td><td>M1
A1
M1
A1</td></tr>
<tr><td></td><td></td><td>4</td></tr>
<tr><td>2</td><td>
$$h=0.5$$
<table>
<tr><td>x</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td></tr>
<tr><td>y</td><td>1</td><td>0.7115</td><td>0.5906</td><td>0.5218</td></tr>
</table>

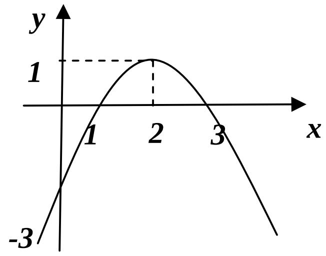
$$\int_1^{2.5}\frac{1}{1+\ln x}\,dx\approx\frac{1}{2}(0.5)\left[1+0.5218+2(0.7115+0.5906)\right]$$
$$\approx 0.25(4.1260)$$
$$\approx 1.03$$
</td><td>B1 for x values
B1 for y values
M1
A1</td></tr>
<tr><td></td><td></td><td>4</td></tr>
<tr><td>3</td><td>
$\alpha+\beta=28,\quad \alpha\beta=16$
$$\left(\sqrt{\alpha}+\sqrt{\beta}\right)^2=\alpha+2\sqrt{\alpha}\sqrt{\beta}+\beta$$
$$=\alpha+\beta+2\sqrt{\alpha}\sqrt{\beta}$$
$$=28+2(\sqrt{16})=36$$
$\sqrt{\alpha}+\sqrt{\beta}=6;$
$\sqrt{\alpha}\sqrt{\beta}=4$
∴ $x^2-6x+4=0$
</td><td>B1 (Both)
M1
A1
M1A1</td></tr>
<tr><td></td><td></td><td>5</td></tr>
</table>

4	Let $w = 1 + yi$ $\frac{1}{w} = \frac{1}{1+yi}$ $= \frac{1}{1+yi} \times \frac{1-yi}{1-yi}$ $= \frac{1-yi}{1+y^2}$ $\text{Re}\left(\frac{1}{w}\right) = \text{Re}\left(\frac{1}{1+yi}\right) = \frac{1}{4}$ $\frac{1}{1+y^2} = \frac{1}{4}$ $1 + y^2 = 4$ $y^2 = 3 \rightarrow y = \pm\sqrt{3}$ $\therefore \quad w = 1 + \sqrt{3}i, \quad w = 1 - \sqrt{3}i$	B1 M1 A1 M1 M1 A1
		6
5	$x^3 + xy + 2y^3 = p$ -------------- (1) $3x^2 + x\frac{dy}{dx} + y + 6y^2\frac{dy}{dx} = 0$ $(x + 6y^2)\frac{dy}{dx} = -(3x^2 + y)$ $\frac{dy}{dx} = \frac{-(3x^2 + y)}{(x + 6y^2)}$ If the curve has a tangent which is parallel to *y*-axis, then $x + 6y^2 = 0$ ---------------------- (2) Substitute $x = -6y^2$ into equation (1), hence $(-6y^2)^3 + (-6y^2)y + 2y^3 = p$ $-216y^6 - 4y^3 = p$ $216y^6 + 4y^3 + p = 0$ Since *y* is real, and write quadratic equation in y^3. $216(y^3)^2 + 4y^3 + p = 0$ $b^2 - 4ac \geq 0$ $(4)^2 - 4(216)(p) \geq 0,$ $\therefore \quad p \leq \frac{1}{54}$	**M1 A1** **A1** **B1** **M1** **A1** **M1 (*y* is real or show b^2-4*ac* $\geq$ 0)** **A1**
		8

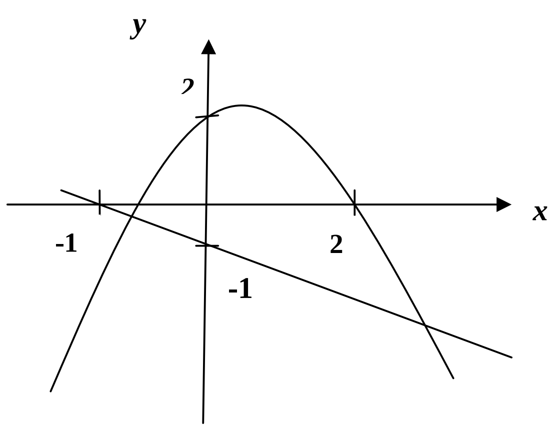
6 (*a*)	$f(x)=x+\frac{1}{x},\quad x\geq 1$ $f'(x)=1-\frac{1}{x^2},\quad \forall x\geq 1$ $x\geq 1\Rightarrow \frac{1}{x^2}\leq 1$ $1-\frac{1}{x^2}\geq 0$ $f'(x)\geq 0$ $\therefore\quad f(x)$ increases	**M1 for $f'(x)$** **M1** **A1**
(*b*)	When $x = 1$, $f(x) = 2$ and $f(x)$ increases as x increases, $f(x)\geq 2$, $\{y : y\geq 2)$	**B1**
(*c*)	Let $y=f^{-1}(x)$ $f(y)=x$ $y+\frac{1}{y}=x$ $y^2-xy+1=0$ $y=\frac{-(-x)\pm\sqrt{(-x)^2-4(1)(1)}}{2(1)}$ $y=\frac{x\pm\sqrt{x^2-4}}{2}$ Hence, $y=\frac{x+\sqrt{x^2-4}}{2},\quad$ for $x\geq 2$ $\therefore\quad f^{-1}(x)=\frac{x+\sqrt{x^2-4}}{2},\quad x\geq 2$	**M1** **M1** **A1** **A1**
		8
7 (*a*)	$(-2)^3 + 2(-2)^2 - 3(-2) - 6 = 0$ ∴ $x + 2$ is a factor of $x^3 + 2x^2 - 3x - 6$.	**M1** **A1**
(*b*)	$x^3 + 2x^2 - 3x - 6 = (x+2)(x^2 - 3)$ $= (x+2)(x+\sqrt{3})(x-\sqrt{3})$ The other two linear factors are $x+\sqrt{3}$ and $x-\sqrt{3}$.	**M1** **M1** **A1**

(*c*)	$\frac{x^3+2x^2-3x-6}{x-1} \le 0$ $\frac{(x+2)(x+\sqrt{3})(x-\sqrt{3})}{x-1} \le 0$ ………… $\{x : -2 \le x \le -\sqrt{3},\quad 1 \le x \le \sqrt{3}\}$	**B1 (2nd line)** **M1** **(Any correct method to obtain answer)** **A1**
		8
8	$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & -2 \\ -1 & 1 & -3 \\ -3 & -2 & 2 \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} 4 & -2 & 1 \\ -11 & 2 & -8 \\ -5 & -1 & -3 \end{pmatrix}$ $\mathbf{AB} = \begin{pmatrix} 2 & 1 & -2 \\ -1 & 1 & -3 \\ -3 & -2 & 2 \end{pmatrix}\begin{pmatrix} 4 & -2 & 1 \\ -11 & 2 & -8 \\ -5 & -1 & -3 \end{pmatrix}$ $= \begin{pmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{pmatrix}$	**B1**
	$\boldsymbol{AB} = 7\boldsymbol{I}$	**B1**
	$A^{-1} = \frac{1}{7}B$	**M1**
	$A^{-1} = \frac{1}{7}\begin{pmatrix} 4 & -2 & 1 \\ -11 & 2 & -8 \\ -5 & -1 & -3 \end{pmatrix}$ $= \begin{pmatrix} \frac{4}{7} & -\frac{2}{7} & \frac{1}{7} \\ -\frac{11}{7} & \frac{2}{7} & -\frac{8}{7} \\ -\frac{5}{7} & -\frac{1}{7} & -\frac{3}{7} \end{pmatrix}$	**A1**
	$2\boldsymbol{x} + \boldsymbol{y} - 2\boldsymbol{z} = 3$ $-2\boldsymbol{x} + 2\boldsymbol{y} - 6\boldsymbol{z} = -14$ $-3\boldsymbol{x} - 2\boldsymbol{y} + 2\boldsymbol{z} = -5$ $\begin{pmatrix} 2 & 1 & -2 \\ -2 & 2 & -6 \\ -3 & -2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -14 \\ -5 \end{pmatrix}$	**B1**

	$\begin{pmatrix} 2 & 1 & -2 \\ -1 & 1 & -3 \\ -3 & -2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -7 \\ -5 \end{pmatrix}$	**M1**
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 4 & -2 & 1 \\ -11 & 2 & -8 \\ -5 & -1 & -3 \end{pmatrix}\begin{pmatrix} 3 \\ -7 \\ -5 \end{pmatrix}$	**M1**
	$= \frac{1}{7}\begin{pmatrix} 21 \\ -7 \\ 7 \end{pmatrix}$	
	$= \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$	**A1**
	$\therefore \quad x = 3, \quad y = -1, \quad z = 1$	**A1**
		9
9	$9x^2 + 4y^2 - 18x + 16y - 119 = 0$ $9(x^2 - 2x) + 4(y^2 + 4y) - 119 = 0$ $9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) - 25 - 119 = 0$ $9(x-1)^2 + 4(y+2)^2 - 144 = 0$ $9(x-1)^2 + 4(y+2)^2 = 144$ $\frac{(x-1)^2}{16} + \frac{(y+2)^2}{36} = 1$ It is an ellipse, with centre $(1, -2)$ 	**B1 (2^{nd} line)** **M1 (3^{rd} line, completing the square)** **A1 (5^{th} line)** **A1 (6^{th} line)** **A1(conclusion with centre)** **B1 for (1,4) & (1, -8)** **B1 for (-3,-2) & (5, -2)** **D1 (Shape)** **D1 (All correct)**
		9

10	$x = 2 + t \Rightarrow \frac{dx}{dt} = 1$ $y = 1 - t^2 \Rightarrow \frac{dy}{dt} = -2t$	**M1**
	Gradient of tangent is $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -2t$	**A1**
	So, gradient of normal is $\frac{1}{2t}$ Equation of the normal is $y - (1 - t^2) = \frac{1}{2t}[x - (2 + t)]$	**M1**
	$2ty - 2t + 2t^3 = x - 2 - t$ $x - 2ty = 2t^3 - t + 2$	**A1**
	When t = 2, the equation of normal at point T is $x - 2(2)y = 2(2)^3 - (2) + 2$ $x - 4y = 16$	**M1**
	When t = p, the coordinates is $(2 + p, 1 - p^2)$ Since P lies on the normal, then $2 + p - 4(1 - p^2) = 16$	**M1**
	$4p^2 + p - 18 = 0$	**A1**
	$(4p + 9)(p - 2) = 0$ $p = -\frac{9}{4}$, *reject* $p = 2$	**M1**
	the coordinates of P is $[2 + (-\frac{9}{4}), 1 - (-\frac{9}{4})^2] = (-\frac{1}{4}, -\frac{65}{16})$	**A1**
	From $x = 2 + t$; $y = 1 - t^2$ $t = x - 2$, then $y = 1 - (x - 2)^2$ (quadratic function)	**B1**
		**D1** **D1**
		12

11 **(*a*)**	$(1+x)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)x+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}x^2+$ $=\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}x^3+\ldots$	**M1**
	$=1-\frac{1}{2}x+\frac{3}{8}x^2-\frac{5}{16}x^3+\ldots$	**A1**
	Given $x=\frac{1}{8}$	
	$(1+\frac{1}{8})^{-\frac{1}{2}}=\left(\frac{9}{8}\right)^{-\frac{1}{2}}=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}$	**B1**
	$\frac{2\sqrt{2}}{3}=1-\frac{1}{2}\left(\frac{1}{8}\right)+\frac{3}{8}\left(\frac{1}{8}\right)^2-\frac{5}{16}\left(\frac{1}{8}\right)^3+\ldots$	**M1**
	$=\frac{7723}{8192}$	
	$\therefore\ \sqrt{2}=1.41412$	**A1**
(*b*)	$\frac{1}{4r^2-1}=\frac{1}{(2r-1)(2r+1)}$ $=\frac{A}{2r-1}+\frac{B}{2r+1}$	**B1**
	$1=A(2r+1)+B(2r-1)$	
	When $r=\frac{1}{2},\quad 2A=1\Rightarrow A=\frac{1}{2}$	
	When $r=-\frac{1}{2},\quad -2B=1\Rightarrow B=-\frac{1}{2}$	**M1A1**
	$\frac{1}{4r^2-1}=\frac{1}{2}\left[\frac{1}{2r-1}-\frac{1}{2r+1}\right]$	**A1**

(i)	$\sum_{r=1}^{n}\frac{1}{4r^2-1}=\frac{1}{2}\sum_{r=1}^{n}\left[\frac{1}{2r-1}-\frac{1}{2r+1}\right]$ $=\frac{1}{2}\left[\begin{array}{l}(1-\frac{1}{3})+(\frac{1}{3}-\frac{1}{5})+(\frac{1}{5}-\frac{1}{7})+\ldots\\+(\frac{1}{2n-3}-\frac{1}{2n-1})+(\frac{1}{2n-1}-\frac{1}{2n+1})\end{array}\right]$ $=\frac{1}{2}\left[1-\frac{1}{2n+1}\right]$ $=\frac{n}{2n+1}$	B1 M1 A1
(ii)	$\sum_{r=1}^{\infty}\frac{1}{4r^2-1}=\lim_{n\to\infty}\frac{1}{2}\left[1-\frac{1}{2n+1}\right]$ $=\frac{1}{2}$	**B1**
		13
12(*a*) **(i)**		**D1** Must show all intersections **D1** Must show all intersections
(ii)	$2+x-x^2=-x-1$ $x^2-2x-3=0$ $(x-3)(x+1)=0$ $x=3,\quad x=-1$ When $x=3,\quad y=-4$ When $x=-1,\quad y=0$ $\therefore$ the coordinates are $(3,-4),\quad(-1,0)$	**M1** **A1**

(iii)	$Area = \int_{-1}^{3} \left[(2+x-x^2)-(-x-1) \right] dx$	**M1**
	$= \int_{-1}^{3} (3+2x-x^2) dx$	
	$= \left[3x + x^2 - \frac{x^3}{3} \right]_{-1}^{3}$	**A1**
	$= \left[(9+9-9) - (-3+1+\frac{1}{3}) \right]$	**M1**
	$= 10\frac{2}{3} unit^2$	**A1**
(*b*)	$Volume = \pi \int_{1}^{2} (\frac{x+5}{x+1} - 1)^2 dx$	**M1**
	$= \pi \int_{1}^{2} \left(\frac{4}{x+1} \right)^2 dx$	**A1**
	$= 16\pi \int_{1}^{2} (x+1)^{-2} dx$	**M1**
	$= 16\pi \left[-\frac{1}{(x+1)} \right]_{1}^{2}$	**A1**
	$= 16\pi \left[-\frac{1}{3} + \frac{1}{2} \right]$	**M1**
	$= \frac{8\pi}{3} unit^3$	**A1**
		14